state, although this was not noted in the reported results. Solution for the supersonic problem have been attempted.[3] Periodic solutions were not obtained, except for conditions with very weak shocks, as would be expected.

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Theoretical Gain Optimization in CO_2-N_2-H_2 Gasdynamic Lasers with Two-Dimensional Wedge Nozzles

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Introduction

IN an earlier publication,[1] we reported the theoretical gain optimiation studies of a 16-μm CO_2-N_2-H_2 gasdynamic laser (GDL) in which it was showed that the optimum value of a 2.06% cm^{-1} gain could be obtained with conical or hyperbolic nozzles. In this Note, we report the results of optimization studies with a 16-μm CO_2-N_2-H_2 GDL using two-dimensional wedge nozzles. We have found that the optimum value of the gain that can be achieved is as high as 12.7% cm^{-1} on the P(15) line of the $(02^00)\rightarrow(01^10)$ transition of CO_2 for a gas mixture composition of CO_2:N_2:H_2 = 30:50:20%. The corresponding optimum values for the reservoir pressure and area ratio are computed as functions of the resevoir temperature and presented graphically. Results are presented for a range of gas mixture compositions.

Governing Equations

The global mass, momentum, and energy conservation equations governing the steady inviscid quasi-one-dimensional flow of a CO_2-N_2-H_2 gas mixture in a supersonic nozzle of a GDL are considered along with the equation of state. Equations governing the vibrational energy exchange through bimolecular collisions are also considered. Two additional algebraic equations for the population inversion (PI) and the small-signal optical gain (G_0) are obtained as functions of the flow quantities from the basic governing equations. All of these equations (which are in dimensional form) are normalized with respect to a chosen set of reference values and then reduced to universal form by suitable transformation of the independent variable, so that the solutions depend on a single universal parameter χ_I that combines all of the other parameters in the system. The governing equations and the equations for PI and G_0 in their final form are given in Ref. 1.

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The following functions are used for solving the equations in the present case.

1) The nonsimilar function:

$$N_s = 0.0, \qquad \text{for } (\xi_*/\xi) < 0.980$$
$$= 0.03 - 12.5(0.990 - \xi_*/\xi)^{1.208} \qquad \text{for } 0.980 \leq (\xi_*/\xi) < 0.990$$
$$= 0.2747 - 1.676\times10^{-2}(\xi_*/\xi - 0.96)^{-0.85} \qquad \text{for } (\xi_*/\xi) \geq 0.990 \tag{1}$$

where ξ is an independent parameter.

2) The mass flow factor $(\rho_* u_*)$ and the density ρ_*:

$$\rho_* u_* = 0.669 = \text{const} = K_1 \tag{2}$$

$$\rho_* = 0.6338 = \text{const} = K_2 \tag{3}$$

3) The normalized velocity ratio (u/u_*):

$$(u/u_*)K_1^{8.08} = -0.0036 + 0.0635(0.650 + \log_{10}A)^{-3.176} \qquad \text{for } M < 1.0$$
$$= 0.5937 - 0.13789(0.250086 + \log_{10}A)^{-0.7194} \qquad \text{for } M \geq 1.0 \tag{4}$$

where A is the normalized area ratio, M the Mach number, and subscript (*) the throat conditions.

Results

Following the numerical scheme given in Ref. 2, the governing equations are solved for the normalized values of the translational temperature ψ and the vibrational temperatures ϕ_I and ϕ_{II} of modes I and II, respectively, for a family of wedge nozzles ($ij = 1.0$) with χ_I as a parameter. Computations are carried out for a range of mixture compositions, for a H_2 mole fraction having a 5-20% variation with a CO_2 mole fraction variation of 2.5-30% for each H_2 mole fraction. Variations in ψ, ϕ_I, and ϕ_{II} along the nozzle are computed for each case and used to compute the corresponding values of PI and G_0.

A typial variation of these solutions for a sample case of CO_2:N_2:H_2 = 20:73:7% are presented in Fig. 1, where gain G_0 attains a maximum, while the PI remains constant far downstream of the throat. Such peak values for G_0 are computed for a range of mixture compositions and over a range of χ_I values for each composition. The plots of maximum values of G_0 vs χ_I are shown in Fig. 2 for different CO_2-N_2 concentrations with 20% of H_2 mole fraction. This particular set is presented because the optimum gain shown is the highest in the wide range of mixture compositions for which the optimum gain values are computed. From these graphs, it is clear that, for every gas mixture, G_0 attains a maximum, designated as $(G_0)_{opt}$, at a particular value of χ_I, designated as $(\chi'_I)_{opt}$ and representing the optimum value. Thus, for a given laser gas mixture composition, $(G_0)_{opt}$ represents the highest possible value of small-signal gain that can be achieved. For example, the $(G_0)_{opt}$ equal to 12.7% cm^{-1} occurs at a $(\chi_I)_{opt}$ value of 2.5 for a gas mixture composition of CO_2:N_2:H_2 = 30:50:20% as shown in Fig. 2. The variation of $(G_0)_{opt}$ is shown in Fig. 3 for various CO_2 and H_2 concentrations.

Optimum operating conditions, such as the reservoir temperature and pressure corresponding to the optimum value of G_0 for any given laser mixture, can be readily calculated using the corresponding $(\chi_I)_{opt}$ value. Since the universal parameter χ_I is a function of the reservoir temperature T_0' and

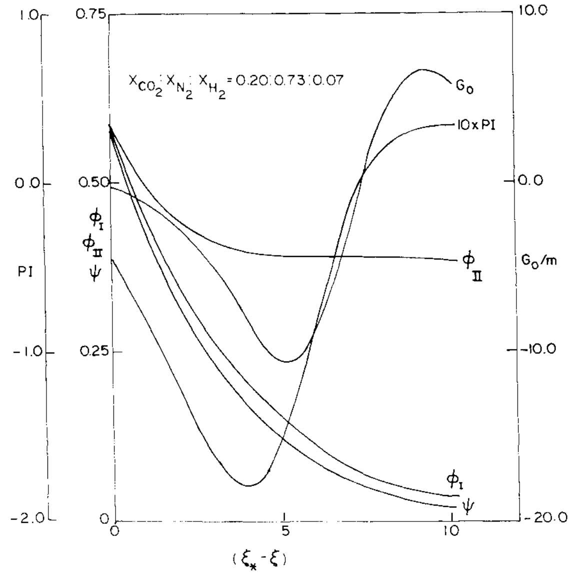

Fig. 1 Variation of flow quantities for $\chi_I = 4.0$, $\xi_* = 28.588$.

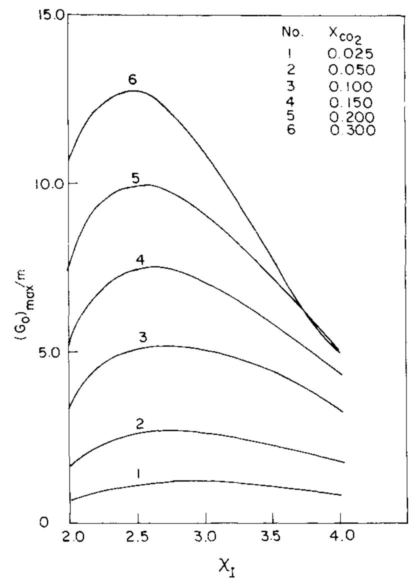

Fig. 2 Variation of $(G_0)_{max}$ with χ_I.

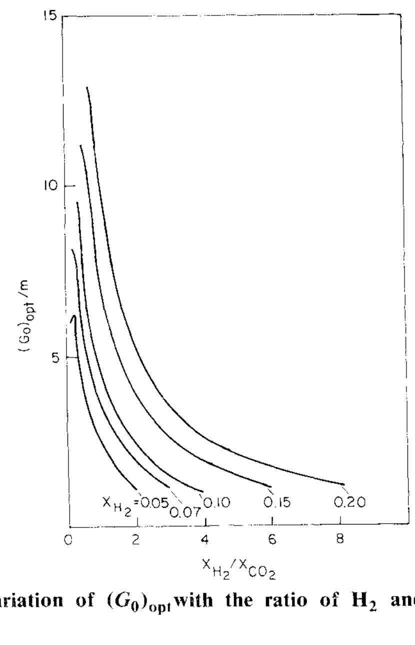

Fig. 3 Variation of $(G_0)_{opt}$ with the ratio of H_2 and CO_2 mole fractions.

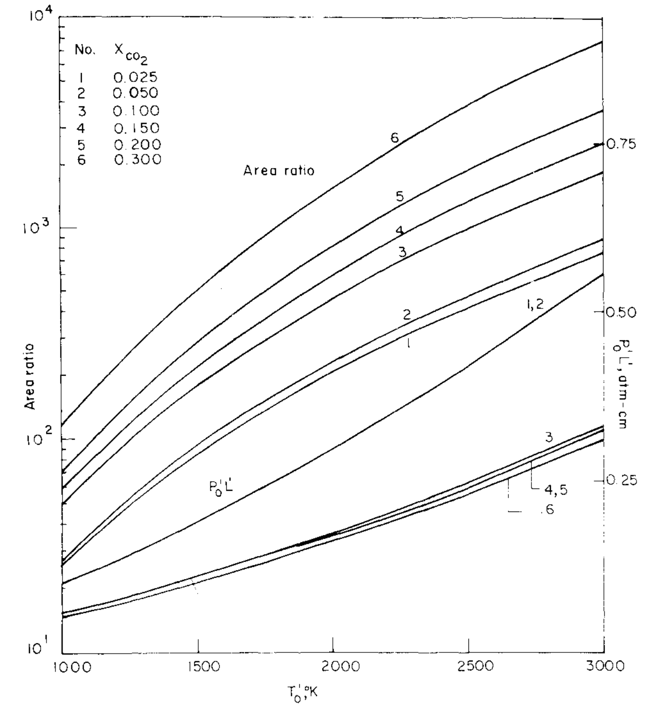

Fig. 4 Variation of optimum area ratio and $P_0'L'$ with T_0' for $X_{H_2} = 0.20$.

reservoir pressure P_0' along with the nozzle shape factor L', the value for the binary parameter $P_0'L'$ can be calculated as a function of T_0'. The variations in $P_0'L'$ with T_0' are presented in Fig. 4 in graphic form for a sample set of compositions. These graphs can be used to obtain the optimum P_0' values for any T_0' and for any chosen value of L'.

Finally, the optimum area ratio A_{opt} corresponding to the $(G_0)_{opt}$ value for any given gas mixture is computed using the following equation:

$$[0.5937 - 0.13789(0.250086 + \log_{10}A_{opt})^{-0.7194}]A_{opt} = K_1^{8.08}K_2\exp(S_0 - \xi_{opt}) \tag{5}$$

where K_1, K_2, and S_0 were defined earlier and ξ_{opt} is the value of ξ at which $(G_0)_{opt}$ occurs. Since K_1, K_2, and S_0 are functions of T_0' only, Eq. (5) can be solved for A_{opt} as a function of T_0' for a known value of ξ_{opt}. The variations of A_{opt} with T_0' are presented in the form of graphs in Fig. 4 for a sample set of compositions.

Conclusions

Similar solutions are obtained for the equations governing the 16-μm CO_2-N_2-H_2 gasdynamic laser employing a two dimensional wedge nozzle, which are then further used to obtain optimum values of the small-signal laser gain and the corresponding optimum values of the reservoir pressure, reservoir temperature, and nozzle area ratio for a given gas mixture composition. The results are presented in the form of graphs for a range of laser mixture compositions. We have found that for a gas composition of CO_2:N_2:H_2 = 30:50:20% up to a 12.7% cm^{-1} gain can be achieved on the P(15)(02°0)→(01¹0) transition of CO_2 in 16-μm CO_2-N_2-H_2 GDL system.

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